An Analytic Model of Direct-Contact Heat Transfer in Spray-Column Evaporators

Yasuhiko H. Mori
Dept. of Mechanical Engineering, Keio University, Yokohama 223, Japan

The heat transfer to, and the resultant evaporation of, drops of a volatile liquid sprayed upward in an immiscible liquid flowing down in a vertical column are analyzed to enable calculation of the volumetric heat transfer coefficient in the column. For this analysis, a model is contrived that assumes no nucleation delay in initially monodispersed drops and a heat transfer to each of the drops, with simultaneous evaporation, that can be approximated by an empirical correlation for heat transfer to an isolated drop evaporating in a quiescent, sufficiently extended medium. The expression obtained for the volumetric heat transfer coefficient is used to predict its values under some particular column operating conditions, which are then compared with relevant experimental data found in the literature.

Introduction

Fluid motion and heat transfer in spray columns used as direct-contact evaporators have been studied over the last quarter-century by more than research groups, including Sideman et al. (1965), Sideman and Gat (1966), Blair et al. (1976), Smith et al. (1982), Battya et al. (1983), Shiina and Sakaguchi (1984), Goodwin et al. (1985), Tadrist et al. (1987), Çoban and Boehm (1986, 1989), Seetharamu et al. (1987, 1989), Steiff et al. (1990), and Core and Mulligan (1990). Because of the complexity of the multiphase structure in spray columns, their heat exchange performance has often been expressed in terms of the so-called volumetric heat transfer coefficient, α_v, defined as

$$\dot{Q} = AH\alpha_v \Delta T \quad (1)$$

where $\dot{Q}$ is the rate of heat supply from the continuous phase to the evaporating dispersed phase in a column whose cross-sectional area and effective height for the heat exchange are A and H, respectively. The temperature difference between the two phases, ΔT, may be considerably nonuniform along the column axis. If that is the case, ΔT in Eq. 1 is read as a representative temperature difference such as ΔT_{lm}, the logarithmic mean of the excesses of the inlet and the outlet temperatures of the continuous phase over the saturation temperature of the dispersed-phase fluid corresponding to the pressure in the column. The volumetric heat transfer coefficient in a column can be determined experimentally without regard to the interfacial area held in the column at each instant, but its value thus determined must be specific to the operational condition set in the column in determining the value as well as to the geometry and the dimensions of the column. Thus, it is important to develop a general method that enables us to predict the volumetric heat transfer coefficient, depending only on our common knowledge of fluid mechanics and heat transfer in multiphase systems. The knowledge may be the size of drops sprayed with given orifices or nozzles, and the motion and the heat transfer that an isolated drop of a known size would show during its evaporation in a stagnant, sufficiently extended medium.

The first effort directed along this line of investigation was done by Smith et al. (1982). The analytic model presented by them can take account of possible bubble coalescence and yields an explicit, closed-form expression for the volumetric heat transfer coefficient, α_v. The drawback of the model may be that it assumes a continuous phase that is stagnant and uniform in temperature, a condition satisfied only approximately in limited applications such as residential cool storage systems in batch operation (Carbajo, 1985; Mori and Mori, 1989). Shiina and Sakaguchi (1984) incorporated the model of Smith et al. into their scheme for calculating the performance of a parallel-flow spray column consisting of preheater, evaporator, and superheater sections. More recently, Çoban and Boehm (1986, 1989) reported on a numerical analysis of the performance of a spray-column evaporator. A unidirectional flow of the continuous and the dispersed phases was supposed. The variation in the continuous-phase temperature along the column axis was evaluated on the basis of heat balance at each axial location. No bubble coalescence was assumed. Core and Mulligan (1990) presented a population balance analysis of a batch evaporator, assuming the evaporation of drops in a uniform but transient temperature field and a negligible bubble coalescence.

This paper presents an attempt to derive an expression for the volumetric heat transfer coefficient α_v in a counterflow spray column in the form of a simple function of a few column operating parameters in addition to fluid properties. The expression may improve our insight into the dependencies of the total heat transfer performance of spray columns on individual operational parameters and, indirectly, on the column design.

Analytical Model and Formulation

General description of the model

We suppose a counterflow spray column in which the dispersed phase is injected in the form of saturated liquid drops of a uniform size, resulting in an immediate start of evaporation. The most fundamental assumption in the present model is that the heat transfer to each drop, taking the form of a liquid/vapor two-phase bubble in the course of evaporation, is described by the same heat-transfer correlation as that established for single-drop evaporation (Shimizu and Mori, 1988), provided that we simply substitute a mean relative velocity between the continuous and the dispersed phases in the column, U_r, for the bubble velocity used in the correlation. The correlation is given by

$$Nu = C\, Pe^{1/2} \tag{2}$$

where $Nu = \alpha D/\lambda_C$, $Pe = U_r D/\kappa_C$, and C is a constant determined empirically (Shimizu and Mori, 1988). No mutual coalescence of bubbles during evaporation is assumed.

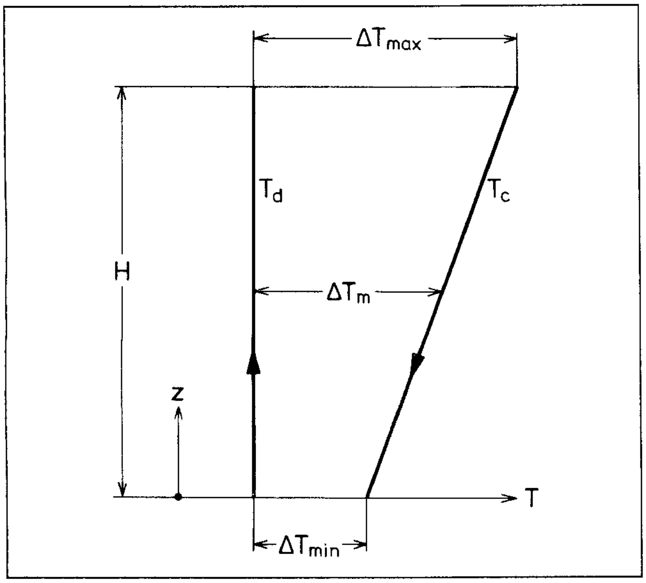

Figure 1. Linear distributions of the continuous-phase temperature, T_c, and temperature difference, ΔT, assumed to be established along column axis.

The axial temperature distribution in the continuous phase is assumed to be linear, Figure 1. This is a crude approximation to the temperature distribution that should be so determined, in principle, as to satisfy the local heat balance condition. In fact, Çoban and Boehm (1986, 1989) determined the temperature distribution on the basis of the local heat balance condition but assuming steady one-dimensional flows with no back mixing and utilizing a numerical solution procedure. The real temperature distribution in a spray column is, however, probably interlinked with more complicated hydrodynamic and transport processes. Considering the inherently approximate nature of the present analysis, the use of an approximation of linear temperature distribution seems to be a reasonable compromise for eliminating the complexity.

Holdup ratio and rise velocity of bubbles

First we define, for convenience, three superficial velocities as

$$u_c = \frac{\dot{V}_c}{A} \tag{3}$$

$$u_d = \frac{\dot{V}_d}{A} = \frac{1}{6}\pi D^3\, nU = \phi U \tag{4}$$

$$u_{dl} = \frac{\dot{M}_d}{\rho_{dl} A} = \frac{1}{6}\pi D_0^3\, nU \tag{5}$$

where $\dot{V}_c$ and $\dot{V}_d$ denote the volumetric flow rate of the continuous phase and that of the dispersed phase, respectively; D is the diameter of each liquid/vapor two-phase bubble; n is the number density of two-phase bubbles; U is the velocity of rise of each bubble relative to a coordinate fixed to the column; ϕ is the holdup ratio of the dispersed phase; $\dot{M}_d$ is the mass flow rate of the dispersed phase; and ρ_{dl} is the mass density of the dispersed phase in the saturated liquid state. According to Richardson and Zaki (1954), the relative velocity, U_r, is related to u_c, u_d, ϕ, and U_t, the terminal velocity that each bubble would have when $u_c \rightarrow 0$ and $n \rightarrow 0$, as

$$U_r = \frac{u_d}{\phi} + \frac{u_c}{1-\phi} = U_t\,(1-\phi)^b \tag{6}$$

where

$$\left.\begin{array}{ll} b = 4.45 Re^{-0.1} - 1 & (1 < Re < 500) \\ b = 1.39 & (Re \gtrsim 500) \end{array}\right\} \tag{7}$$

From Eqs. 4 and 6 we have

$$U = \frac{u_d}{\phi} = U_t\,(1-\phi)^b - \frac{u_c}{1-\phi} \tag{8}$$

or

$$\omega \equiv \frac{U_r}{U} = \left[1 - \frac{u_c}{U_t(1-\phi)^{b+1}}\right]^{-1} \tag{9}$$

Here we assume, in view of the experimental results of Shimizu and Mori (1988), that U_t is held constant during the evaporation of each drop. Besides, we set b at 1.39, considering Re to be of the order of several hundreds or greater during almost the whole process of evaporation, in the medium of water or brine, of a single hydrocarbon or fluorocarbon drop with D_0 exceeding 1.4 mm (Shimizu and Mori, 1988; Shimaoka and Mori, 1990).

Finally we intend to replace ϕ, the residual variable in Eqs. 6, 8, and 9, with a constant possibly representing its mean in the column, $\bar{\phi}$. Combining Eqs. 4 and 5 gives

$$\phi = \left(\frac{D}{D_0}\right)^3 \frac{u_{dl}}{U} \tag{10}$$

If we substitute Eq. 8 into Eq. 10, we get

$$\phi = \left(\frac{D}{D_0}\right)^3 \frac{u_{dl}}{U_t(1-\phi)^{1.39} - u_c/(1-\phi)} \tag{11}$$

Solving Eq. 11 for ϕ by use of an iterative calculation and integrating it numerically between $(D/D_0)^3 = 1$ and $(D/D_0)^3 = \rho_{dl}/\rho_{dv}$ should lead to an evaluation of $\bar{\phi}$. In view of the approximate nature of the analysis under consideration and also the role of $\bar{\phi}$ being no more than secondary, a truncated expression for $\bar{\phi}$ seems enough for the present use, which is given, for example, by

$$\bar{\phi} = \frac{1}{2}\left(\frac{\rho_{dl}}{\rho_{dv}}\right)\left(\frac{u_{dl}}{U_t - u_c}\right) \tag{12}$$

Applying the three approximations listed above to Eqs. 6, 8, and 9, we have the following equations that yield constant values for U_r, U, and ω:

$$U_r = U_t(1-\bar{\phi})^{1.39} \tag{13}$$

$$U = U_t(1-\bar{\phi})^{1.39} - \frac{u_c}{1-\bar{\phi}} \tag{14}$$

$$\omega = \left[1 - \frac{u_c}{U_t(1-\bar{\phi})^{2.39}}\right]^{-1} \tag{15}$$

Derivation of local and average volumetric heat transfer coefficients

The local, volumetric heat transfer coefficient at an elevation z above the nozzle outlet, where the diameter of each bubble arrives at D, is written as

$$\alpha_{v,z} = n\pi D^2 \alpha \tag{16}$$

where α denotes the surface heat transfer coefficient to be specified by Eq. 2 as

$$\alpha = C\lambda_c \kappa_c^{-1/2} U_r^{1/2} D^{-1/2} \tag{17}$$

Substitution of Eq. 17 into Eq. 16 yields

$$\alpha_{v,z} = n\pi C\lambda_c \kappa_c^{-1/2} U_r^{1/2} D^{3/2} \tag{18}$$

The variable D in Eq. 18 needs to be replaced by z. For this purpose, we consider the heat balance for each bubble as

$$\pi D^2 \alpha \Delta T dt = \rho_{dv} h_{lv,d} \frac{\pi D^2 dD}{1 - \rho_{dv}/\rho_{dl}} \approx \rho_{dv} h_{lv,d}\, \pi D^2 dD \tag{19}$$

Substituting Eq. 17 into the lefthand side of Eq. 19 and using the relation $U dt = dz$, we get

$$D^{1/2} dD = \frac{C\lambda_c \Delta T \omega}{\kappa_c^{1/2} \rho_{dv} h_{lv,d} U_r^{1/2}}\, dz \tag{20}$$

Here we introduce the assumption of linear distribution of ΔT as illustrated in Figure 1. It is written as

$$\Delta T = \Delta T_{min} + (\Delta T_{max} - \Delta T_{min}) \frac{z}{H} \tag{21}$$

or, in dimensionless form, as

$$\theta = \theta_{min} + (\theta_{max} - \theta_{min}) \frac{z}{H} \tag{22}$$

where

$$\theta = \frac{\Delta T}{\Delta T_m} \tag{23}$$

$$\Delta T_m = \frac{(\Delta T_{max} + \Delta T_{min})}{2} \tag{24}$$

Substitution of Eqs. 22-24 into Eq. 20 yields

$$D^{1/2} dD = CJa_m \left(\frac{\kappa_c}{U_r}\right)^{1/2} \omega \left[\theta_{min} + (\theta_{max} - \theta_{min}) \frac{z}{H}\right] dz \tag{25}$$

where $Ja_m = \rho_c c_{pc} \Delta T_m / \rho_{dv} h_{lv,d}$. Integrating Eq. 25, we have

$$D^{3/2} - D_0^{3/2} = \frac{3}{2} CJa_m \left(\frac{\kappa_c}{U_r}\right)^{1/2} \times \omega \left[\theta_{min} z + \frac{1}{2}(\theta_{max} - \theta_{min}) \frac{z^2}{H}\right] \tag{26}$$

Equation 26 is then introduced into Eq. 18 to express $\alpha_{v,z}$ as a function of z instead of D:

$$\alpha_{v,z} = n\pi C\lambda_c \kappa_c^{-1/2} U_r^{1/2} \left\{\frac{3}{2} CJa_m \left(\frac{\kappa_c}{U_r}\right)^{1/2} \times \omega \left[\theta_{min} z + \frac{1}{2}(\theta_{max} - \theta_{min}) \frac{z^2}{H}\right] + D_0^{3/2}\right\} \tag{27}$$

The bubble number density n can be eliminated, using the u_{dl}-n relation given in Eq. 5. The resultant expression for $\alpha_{v,z}$ is, if written in dimensionless form:

$$Nu_{v,z}=6C\frac{u_{dl}}{U}\left\{Pe_0^{1/2}+\frac{3}{2}CJa_m\omega\frac{z}{D_0}\times\left[\theta_{min}+\frac{1}{2}(\theta_{max}-\theta_{min})\frac{z}{H}\right]\right\} \quad (28)$$

where $Nu_{v,z} = \alpha_{v,z} D_0^2/\lambda_c$ and $Pe_0 = U_r D_0/\kappa_c$.

The average volumetric heat transfer coefficient in the column, which is defined by Eq. 1, is related to $\alpha_{v,z}$ as

$$\alpha_v=\frac{1}{H\Delta T_m}\int_0^H\alpha_{v,z}\Delta T dz=\frac{1}{H}\int_0^H\alpha_{v,z}\theta\, dz \quad (29)$$

(For an obvious reason, we have substituted ΔT_m, instead of ΔT_{lm}, for ΔT in Eq. 1, the reference temperature difference in the column.) Substitution of Eq. 28 into Eq. 29, followed by some rearrangement, yields

$$Nu_v=6C\frac{u_{dl}}{U}\left(Pe_0^{1/2}+\frac{3}{4}CJa_m\,\omega\frac{H}{D_0}\right) \quad (30)$$

where $Nu_v = \alpha_v D_0^2/\lambda_c$.

Volumetric heat transfer coefficients at critical condition for complete evaporation

Equation 30 permits us to calculate α_v, once we know $\dot{M}_d$ (or u_{dl}), H, and ΔT_m (or Ja_m) as well as the physical properties of the fluids used. However, there exists an as yet unspecified interrelation between $\dot{M}_d$, H, and ΔT_m. In the following, we introduce the interrelation for the particular case $H = H_v$, the minimum height required for the complete evaporation of the dispersed phase.

For the critical condition $H = H_v$, the interrelation between $\dot{M}_d$, H, and ΔT_m is specified by

$$\dot{M}_d h_{lv,d}=AH_v\alpha_v\Delta T_m \quad (31)$$

or, in dimensionless form, by

$$Ja_m\frac{H_v}{D_0}=\frac{\rho_{dl}}{\rho_{dv}}\frac{u_{dl}}{U\omega}\frac{Pe_0}{Nu_v} \quad (32)$$

Substitution of Eq. 32 into Eq. 30, where H is read as H_v, yields

$$Nu_v^2-C_1Nu_v+C_2=0 \quad (33)$$

where

$$C_1=6C\frac{u_{dl}}{U}Pe_0^{1/2}$$

$$C_2=\frac{9}{2}C^2\frac{\rho_{dl}}{\rho_{dv}}\left(\frac{u_{dl}}{U}\right)^2 Pe_0$$

Solving the above quadratic equation for Nu_v, we obtain

$$Nu_v=3C\frac{u_{dl}}{U}Pe_0^{1/2}\left[1+\left(1+\frac{1}{2}\omega\frac{\rho_{dl}}{\rho_{dv}}\right)^{1/2}\right] \quad (34)$$

Combining Eq. 34 with Eqs. 12-15, we can calculate α_v at the critical condition for complete evaporation of the dispersed phase as long as we know u_{dl}, u_c, U_t, and D_0 in addition to the physical properties of the fluids. While α_v thus determined is based on ΔT_m, which we have defined as the reference temperature difference in the column, it is readily converted into the average volumetric heat transfer coefficient based on ΔT_{lm}, when multiplied by the following factor:

$$C_{lm}=\frac{1}{2}\left(\frac{1+\Theta}{1-\Theta}\right)ln\left(\frac{1}{\Theta}\right) \quad (35)$$

where $\Theta = \Delta T_{min}/\Delta T_{max}$.

The heat balance condition given by Eq. 32 is used again to deduce an expression for $Nu_{v,z}$ when $H = H_v$. Subject to Eq. 32, the normalized axial coordinate z/D_0 is written as

$$\frac{z}{D_0}=\frac{H_v}{D_0}\frac{z}{H_v}=\frac{\rho_{dl}}{\rho_{dv}}\frac{u_{dl}}{U\omega}\frac{Pe_0}{Nu_vJa_m}\frac{z}{H_v} \quad (36)$$

Substituting Eq. 34 into Eq. 36 to eliminate Nu_v, and then Eq. 36 into Eq. 28 to eliminate z/D_0, we have

$$Nu_{v,z}=6C\frac{u_{dl}}{U}Pe_0^{1/2}\left\{1+\frac{1}{2}\frac{\rho_{dl}}{\rho_{dv}}\left[1+\left(1+\frac{1}{2}\omega\frac{\rho_{dl}}{\rho_{dv}}\right)^{1/2}\right]^{-1}\times\left[\theta_{min}+\frac{1}{2}\left(\theta_{max}-\theta_{min}\right)\frac{z}{H_v}\right]\frac{z}{H_v}\right\} \quad (37)$$

This equation is not convenient for use, however, because it still involves on its righthand side θ_{max} and θ_{min}. They are eliminated in the next section, assuming an ideal case of no heat loss to the surroundings from the column.

Critical condition for complete evaporation in a thermally insulated column

Here we suppose a spray column well insulated from outside and operated at the critical condition, $H = H_v$. The energy conservation in this case can be written as

$$\dot{M}_c c_{pc}\,(T_{max}-T_{min})=AH_v\alpha_v\Delta T_m \quad (38)$$

where $\dot{M}_c$ is the mass flow rate of the continuous phase. Combining Eq. 38 with Eqs. 23 and 24, we have the following relations:

$$\Delta T_{max}=\Delta T_m\left(1+\frac{AH_v\alpha_v}{2\dot{M}_c c_{pc}}\right) \quad (39)$$

$$\left.\begin{aligned}\theta_{max}&=1+\frac{AH_v\alpha_v}{2\dot{M}_c c_{pc}}=1+\frac{1}{2}\frac{H_v}{D_0}\frac{1}{Pe_0}\frac{U_r}{u_c}Nu_v\\ \theta_{min}&=1-\frac{AH_v\alpha_v}{2\dot{M}_c c_{pc}}=1-\frac{1}{2}\frac{H_v}{D_0}\frac{1}{Pe_0}\frac{U_r}{u_c}Nu_v\end{aligned}\right\} \quad (40)$$

Some of the uses of these relations are exemplified below.

Substituting α_v given by Eq. 34 back into Eq. 31, we can predict the relation between H_v and ΔT_m. However, the temperature difference available to one who is in a position to predict the performance of an evaporator is probably ΔT_{max} rather than ΔT_m. With the aid of Eq. 39, one can estimate H_v relevant to a given ΔT_{max} to be imposed in the evaporator.

If we substitute Eq. 40 into Eq. 37, we can eliminate θ_{max} and θ_{min} from the latter. After some further arrangement utilizing Eq. 34, Eq. 37 becomes

$$Nu_{v,z} = 6C \frac{u_{dl}}{U} Pe_0^{1/2} \left[\!\left[1 + \frac{1}{2} \frac{\rho_{dl}}{\rho_{dv}} \frac{z}{H_v} \left\{ \left[1 + \left(1 + \frac{1}{2} \omega \frac{\rho_{dl}}{\rho_{dv}} \right)^{1/2} \right]^{-1} - \frac{3}{2} C \frac{u_{dl}}{u_c} \omega Pe_0^{-1/2} \frac{H_v - z}{D_0} \right\} \right]\!\right] \tag{41}$$

This equation permits us to predict, without referring to the distribution of temperature differences, the distribution of $\alpha_{v,z}$ to be established along the column axis.

Discussion

The expressions for Nu_v and $Nu_{v,z}$ derived above suggest that the volumetric heat transfer coefficient is inversely proportional to the initial drop diameter, D_0, to approximately the 3/2 power. Thus, a knowledge of D_0 is indispensable to prediction of the volumetric heat transfer coefficient in any system. This fact poses a serious problem in evaluating the validity of the present analytic model on the basis of previously reported, experimentally determined values of the average volumetric heat transfer coefficient. This is because most such experimental results are not accompanied by any statement about observation of initial drop diameters. Despite a mass of literature on the prediction of the size of drops formed at orifices or nozzles of a given diameter, we have still no means usable with complete confidence for predicting the drop size even in an isothermal system with minimal convection or turbulence in the surrounding medium. In a typical direct-contact evaporator, drop formation may be affected by the flow of the surrounding liquid and also by the inception of vapor bubble growth in each drop before its detachment from the tip of an orifice or a nozzle. These factors can considerably reduce the mass of each drop released into the medium of the hotter liquid, and thus cause great difficulty in predicting even roughly the initial drop diameter D_0 (which may either be the actual diameter of each liquid drop just detached or the diameter of a liquid drop of mass equivalent to each detached drop involving a vapor phase from the first). A considerable uncertainty in the initial drop diameter in the experiments inevitably yields an ambiguity in the following comparison between the predictions and the experimental values of the average volumetric heat transfer coefficient.

Reference experimental data

In the literature we find three sources of experimental data that are considered, with a reasonable accuracy, to be values of α_v at the critical condition for complete evaporation. These sources are the work by Sideman and Gat (1966), Smith et al. (1982), and Seetharamu and Battya (1989). Sideman and Gat obtained data on α_v in a spray column in which *n*-pentane was sprayed into a counterflow of water. Of particular interest here are the three data points (runs B1, B2, and B3) each representing nearly the maximum values of α_v observed at a particular flow rate of the dispersed phase, $\dot{M}_d$, while the flow rate of the continuous phase, $\dot{M}_c$, was fixed. These data are assumed to be values of α_v at the condition $H \approx H_v$, and are replotted in Figure 2 for comparison with the predictions. Smith et al. and Seetharamu and Battya employed similar experimental schemes. Smith et al. (1982) observed, in each experimental run, the cooling of stagnant water in a column while cyclopentane was being sprayed to evaporate in the water, and found a critical value of ΔT, the difference between the uniform water temperature and the saturation temperature of cyclopentane, at which the vaporization of cyclopentane just became incomplete. From each $\Delta T - H_v$ relation thus determined, one can readily calculate α_v at the the critical condition $H = H_v$. Shown in Figure 3 are the data obtained under the condition that cyclopentane was sprayed through seven 0.5 mm dia. holes at $\dot{V}_d = 6.31 \times 10^{-6}$ m^3/s. Seetharamu and Battya (1989) used *n*-pentane or R 113 as the dispersed phase to be sprayed into stagnant water. The data obtained with *n*-pentane sprayed through nineteen and thirty-six 0.5 mm dia. holes at $\dot{M}_d = 1$ g/s are plotted in Figure 4.

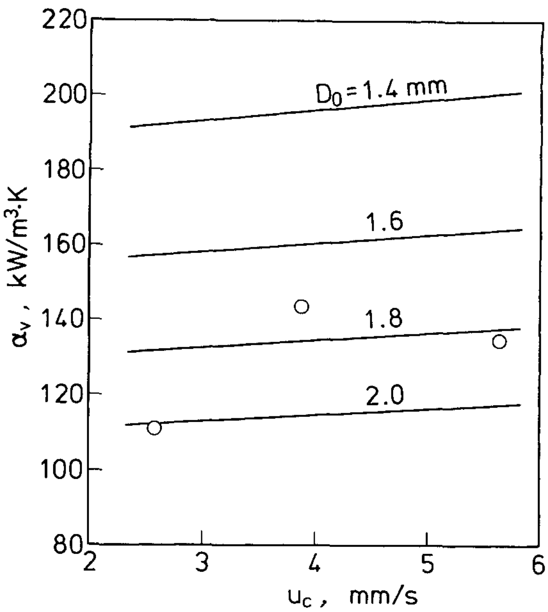

Figure 2. Comparison between predictions and experimental results.

—— prediction
○ Sideman and Gat (1966)
$\dot{M}_d$ = 0.93 g/s, ΔT_{min} = 3.0 ± 0.3 K

It is very important to note that all experimental data on α_v provided in the original reports (Sideman and Gat, 1966; Smith et al., 1982; Seetharamu and Battya, 1989) are based on the assumption that T_d, the constant dispersed-phase temperature, is given by T_{ds}, the saturation temperature of the dispersed-phase substance at the system pressure ($\approx$ 101.3 kPa). Shimizu and Mori (1988) indicated, however, a practical advantage of defining T_d as a temperature T_s at which the sum of the vapor pressures of the dispersed-phase substance and the continuous-phase substance (water) is equal to the system pressure. The values of T_{ds} and T_s in the *n*-pentane/water and cyclopentane/water systems are listed in Table 1. Shimizu and Mori (1988) and Shimaoka and Mori (1990) provided the values of C for

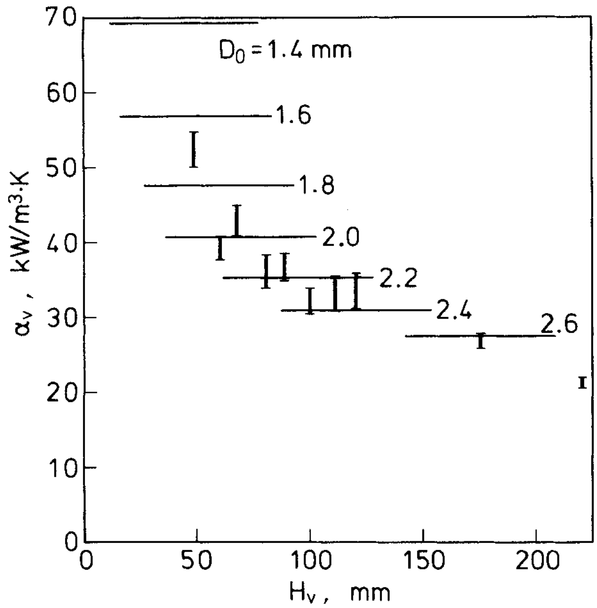

Figure 3. Comparison between predictions and experimental results.

—— prediction
I Smith et al. (1982)
$\dot{V}_{di}$ = 6.31 × 10^{-6} m³/s

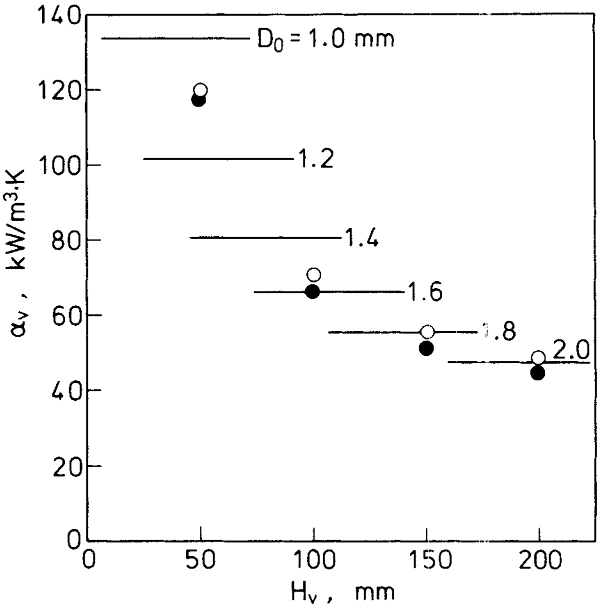

Figure 4. Comparison between predictions and experimental results.

—— prediction
○ 19 injection holes, ● 36 injection holes (Seetharamu and Battya, 1989; Battya, 1983)
M_d = 1 g/s

the *n*-pentane/water system exclusively based on the definition $\Delta T = T_c - T_s$. For consistency in using these C values, the present author has recalculated the logarithmic mean temperature differences or the uniform temperature differences, replacing T_{ds} with T_s, assumed to be set in the experiments of Sideman and Gat, Smith et al., and Seetharamu and Battya, and has thereby redetermined the values of α_v. The data points plotted in Figures 2-4 represent the values of α_v thus redetermined.

Predictions based on the analysis

The predictions indicated in Figures 2-4 are based on Eq. 34 and also Eqs. 12-15. The physical properties of *n*-pentane are evaluated at T_{ds} (= 36.07°C) corresponding to the atmospheric pressure (101.3 kPa), being based on data given in the *TRC Thermodynamic Tables* (1989). The properties of water coupled with *n*-pentane are evaluated at a temperature higher than T_{ds} by 5 K, being based on data given in the *JSME Steam Tables* (1980) and the *JSME Data Book* (1983). For the cyclopentane/water system, the properties tabulated in the paper of Smith et al. (1982) are employed as given. Several different initial drop diameters are assumed arbitrarily in each set of predictions, considering the uncertainty of the drop size in the corresponding experiments. According to Shimizu and Mori (1988), C is set at 0.169, although that may be a slight underestimation for drops larger than 2 mm in initial diameter (Shimaoka and Mori, 1990).

Comparison of predictions with experimental data

Sideman and Gat stated that *n*-pentane issued from 0.5 mm dia. holes at a velocity of 0.65-0.70 m/s and disintegrated into drops of approximately 1.4 mm dia. Figure 2 shows that the predictions of α_v for D_0 = 1.4 mm are appreciably higher than the experimental values. The discrepancy may be ascribed in part to a finite delay of nucleation in drops, which was actually observed by Sideman and Gat. Inevitable coalescence of rising two-phase bubbles in the experiments can be another reason for the experimental α_v values being lower than the predicted values. (The predicted values of α_v indicated in Figure 2 are still raised by 6.5-13%, if they are multiplied by the conversion factor C_{lm} defined in Eq. 35.)

The experimental data of Smith et al., in their arrangement, suggest that $H_v\Delta T$ is nearly constant and hence α_v is almost independent of ΔT or H_v. This forms an apparent contrast to Seetharamu and Battya's α_v data, which show strong positive and negative dependencies on ΔT and H_v, respectively. However, alteration of the definition of ΔT as stated above significantly changes the apparent $\alpha_v - \Delta T$ and $\alpha_v - H_v$ relations. With the definition of ΔT as $(T_c - T_s)$, even the data of Smith et al. show appreciable positive and negative dependencies of α_v on ΔT and H_v, respectively, as indicated in Figure 3. Seetharamu and Battya's data rearranged with the same definition of ΔT show even stronger dependencies on ΔT and H_v. Since C in Eq. 2 is nearly constant (Shimizu and Mori, 1988) or negatively, though rather weakly, dependent on ΔT (Shimaoka

Table 1. Saturated Temperatures at 101.325 kPa

System	T_{ds}* °C	T_s** °C
n-Pentane/water	36.065	34.452
Cyclopentane/water	49.262	44.700

*Data given in *TRC Tables* (1989)
**Calculated by Antoine equation with relevant constants given in *TRC Tables* (1989)

and Mori, 1990), as long as α and hence C are evaluated on the basis of the above definition of ΔT, the characteristics of heat transfer to individual two-phase bubbles never contribute, by themselves, to the positive dependency of α_v on ΔT. We need to find some other factors that possibly induce an excessive increase in α_v with an increase in ΔT, or alternatively a reduction in α_v with the decrease in ΔT.

In the experiments of Smith et al. that yielded the data given in Figure 3, cyclopentane was injected at an extremely high velocity ($\approx$ 4.6 m/s) from 0.5 mm dia. holes. Therefore, the cyclopentane was presumably atomized into drops with a wide spread in size. Smith et al. stated that the fraction of drops that failed to nucleate increased with a decrease in ΔT (i.e., an increase in H_v) and that the diameter of such nuclei-free drops was about 1.0 mm. The nuclei-free drops could not be vaporized during their ascent unless they coalesced with evaporating drops, and thus they must have contributed to increasing H_v and thereby decreasing α_v. It should also be noted that H_v measured experimentally may represent the height required for complete evaporation of the largest drops, instead of the average size drops, generated by the atomization. (Although Smith et al. did not state so, the drops that succeeded in nucleation were possibly larger, on the average, than those that failed to nucleate.) Therefore, it is not necessarily surprising that the experimental values of α_v are so low as to be comparable to the predictions for rather large drops ($D_0 \gtrsim$ 1.6 mm).

The injection velocities of *n*-pentane through nineteen and thirty-six 0.5 mm dia. holes in Seetharamu and Batttya's experiments (1989) are estimated to be 0.44 and 0.22 m/s, respectively. Thus, it is reasonable to assume that the initial drop diameters were appreciably larger than 1.4 mm, the diameter observed by Sideman and Gat (1966) at injection velocities of 0.65–0.70 m/s. (The semiempirical correlations by Scheele and Meister [1968] and by de Chazal and Ryan [1971] predict drop diameters as large as 2.7–3.7 mm at injection velocities of 0.22–0.44 m/s.) According to experimental observation, however, α_v tends to increase, exceeding the predicted level of α_v for D_0 = 1.4 mm as ΔT increases and hence H_v decreases. As ΔT increases, it becomes more probable that a vapor phase grows in each drop before its detachment from an injection hole, causing an accelerated rapid increase in the buoyant force acting on the drop and consequently an earlier detachment of the drop that has a smaller mass than that of a vapor-phase-free liquid drop to be detached from the same hole. If this is the case with Seetharamu and Battya's experiments, the observed dependency of α_v on H_v (or ΔT) does not necessarily disagree with the theoretical predictions. Another possible cause of negative dependency of α_y on H_v is the coalescence of two-phase bubbles during their ascent. The fraction of bubbles that pass the distance H_v without making any coalescence with other bubbles should decrease with an increase in H_v, which necessarily results in a decrease of α_v with an increase in H_v.

Throughout the comparisons discussed above, we have found neither definite contradictions between the present analytic model and existing experimental results nor any evident verification of the model. This ambiguity results from insufficient experimental information on the initial drop diameter, the nucleation in drops, and the coalescence of two-phase bubbles. However, we should not overlook some experimental findings that cannot be interpreted on the basis of the present model. Some of the experimental results obtained in early work of Sideman et al. (1965) indicate a much weaker negative dependency of α_v and D_0 than the present model does. Even a positive dependency of α_v on the injection hole diameter, on which D_0 should be positively dependent, is indicated by a piece of experimental results shown by Seetharamu and Battya (1989). These observations, contradictory to the present model, are left for further experimental and theoretical examinations in the future. At present, the author recommends rather conservative use of the present model. That is, its use should be limited to the case that nearly uniform size drops of a nearly saturated liquid, preferably containing vapor embryos, are released in a column at such a moderate frequency as to cause no more than a low to medium holdup ratio ($\lesssim$ 10 %) in the column. A guide to estimating the possibility that each drop contains at least one vapor embryo is given elsewhere (Tochitani et al., 1975).

Acknowledgment

The assistance of N. Ehara in preparing physical property data used in this work is gratefully acknowledged.

Notation

A = cross-sectional area of spray column
b = constant, Eq. 7
c_p = specific heat capacity
C = constant, Eq. 2
C_{lm} = coefficient of conversion between ΔT_m-based α_v and ΔT_{lm}-based α_v
D = equivalent spherical diameter of drop, two-phase bubble, or vapor bubble
D_0 = initial value of D
h_{lv} = specific latent heat of vaporization
H = column height effective for heat exchange
H_v = critical value of H for complete vaporization of dispersed phase
Ja_m = $\rho_c c_{pc} \Delta T_m / \rho_{dv} h_{lv,d}$
$\dot{M}$ = mass flow rate
n = local number density of two-phase bubbles
Nu = $\alpha D/\lambda_c$
Nu_v = $\alpha_v D_0^2/\lambda_c$
$Nu_{v,z}$ = $\alpha_{v,z} D_0^2/\lambda_c$
Pe = $U_r D/\kappa_c$
Pe_0 = $U_r D_0/\kappa_c$
$\dot{Q}$ = rate of heat transfer in spray column
Re = $U_t D/\nu_c$
t = time
T_c = undisturbed temperature in continuous phase
T_d = reference temperature of dispersed phase
T_{ds} = saturation temperature of dispersed-phase substance
T_s = temperature at which sum of saturated vapor pressures of continuous- and dispersed-phase substances equals system pressure
ΔT = $T_c - T_d$
u = superficial velocities, Eqs. 3–5
U = rise velocity of bubble relative to a coordinate fixed to spray column
U_r = relative velocity between continuous and dispersed phases
U_t = terminal velocity of isolated bubble
$\dot{V}$ = volume flow rate
z = height above the location of drop formation

Greek letters

α = instantaneous heat transfer coefficient based on surface area of spherical two-phase bubble
α_v = average volumetric heat transfer coefficient

$\alpha_{v,z}$ = local volumetric heat transfer coefficient
θ = $\Delta T/\Delta T_m$
Θ = $\Delta T_{min}/\Delta T_{max}$
κ = thermal diffusivity
λ = thermal conductivity
ν = kinematic viscosity
ρ = mass density
ϕ = local holdup ratio of dispersed phase
$\bar{\phi}$ = average holdup ratio of dispersed phase
ω = U_r/U

Subscripts

c = continuous phase or continuous-phase substance
d = dispersed phase
dl = dispersed-phase substance in saturated liquid state
dv = dispersed-phase substance in saturated vapor state
lm = logarithmic mean
m = arithmetic mean
max = maximum
min = minimum

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Manuscript received Sept. 7, 1990, and revision received Jan. 15, 1991.